IOSR Journal of Engineering (IOSR JEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
PP 57-62

A Review on Experimental Study of Logarithmic Decrement

Suhas S. Jadhav, Sushant S. Jadhav, A. S. Kavitake, A. A. Lawand
(Department of Mechanical Engineering, MESCoE- Pune, India)

Abstract : *Logarithmic Decrement (λ) experiment is generally used to analyze the damping ratio of under-damped systems within the time domain. The (λ) logarithmic decrement method will become less and less accurate as the damping ratio will increase past about 0.5; it will not be applicable at all for a damping ratio which is greater than 1.0 because the system becomes over-damped. Logarithmic decrement can be defined as, the natural logarithm (ln) of the ratio of any two succeeding amplitudes on the either side of the middle way position. Practically, there is always an internal and external resistance due to which the energy possessed by the vibratory body will finally come to rest at its equilibrium state. The amplitudes of vibration initially are greater, and will go on decreasing and finally completely lost with the period of time. The rate of decreasing the amplitude (Rate of decay) mainly is depended on the amount of damping. Hence the diminishing of vibrations with time is called as damping or in other words the resistance offered by the body to any motion of any vibratory system is called as damping.*

Keywords: *Logarithmic decrement, damped oscillations, EduVirtualLab, Computer Simulation, Virtual Lab*

I. Introduction

The logarithmic decrement is a technique that by measuring the contraction of the oscillation amplitude of a damped system, estimates the damping that the system is experiencing. Due to this procedure, a more specific name for this technique would be the Logarithmic Decrement of the Amplitude. In this work, however, the original name will be respected.

The Logarithmic Decrement (LD), may be used to experimentally find the damping an oscillator is experiencing [1-4], then from this damping, the control parameters of the oscillation may be calculated. Here "control parameters" refers to the viscosity of the environment, angular frequency of the oscillator, its maximum amplitude and, its initial phase. This paper reports an enhancement of the virtual lab previously published by this author [1,2]. The algorithm described in the current paper shows how to experimentally extract two new parameters from the computer simulation, specifically the initial phase and the maximum amplitude of the oscillations.

The Logarithmic Decrement is defined as the natural logarithm of the ratio of any two successive extreme displacements in a damped oscillation. Obviously these two maximum amplitudes are separated by a certain time 't', so that:

$$\frac{x_{n+1}}{x_n} = e^{-\lambda t}, \qquad t = t_{n+1} - t_n \qquad (1)$$

I.1. Damped oscillations

Damped oscillations (see Fig. 1) are in general characterized with the fact that each amplitude of oscillation tends to reduce gradually as time elapses. Clearly, with higher damping, the more faster the oscillations shrinking. Depending on general relationship between the natural frequency of the oscillatory body and the damping effect applied, damped oscillations are usually classified as Supercritical, Subcritical and Critical. In this presented review we are dealing with subcritical case, and this case is also reffered as that of under-damped oscillations. Fig. 1 is displaying two models of oscillating systems preferred by physicists, namely, the spring and the pendulum, and in this case, both are immersed in liquid, which provides the viscosity the oscillators are experiencing.

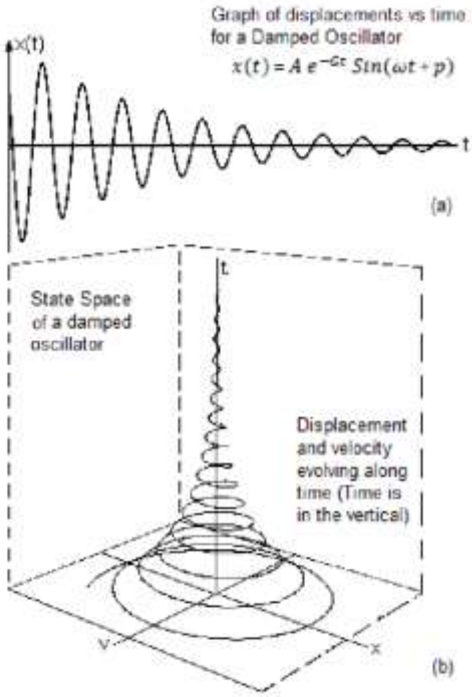

Figure 1: *(a): gradual time development x(t) of any oscillation for a sub-critically damped oscillation, (b): the corresponding State Space, this is the 3D-plotting of displacement and velocity versus time. Notice that the distance between turns in the state space orbit is constant, which means that the oscillations are uniform. In chaotic oscillations these turns are messy and unpredictable.*

I.2. The under-damped oscillator differential equation

From basic vibration physics it is in general known that the differential equation of motion of a system oscillating in presence of a damping is given by,

$$m\frac{d^2\vec{x}}{dt^2} = -k\vec{x} - b\vec{v} \qquad (2)$$

The very first term in the right side of equation (2) is reaction force of any spring (From Hooke's Law) and the second phrase (*bv*) is the viscous damping, shows that as faster the spring oscillates, the higher is the resistance (minus sign) due to the viscosity (*b*) possessed by the medium. Since the velocity is the first temporal derivative of displacement, the scalar derivative form of equation (2) can be written as follows,

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

Here, two coefficients are identified:

$$\omega_o^2 = k/m \qquad G = b/2m \qquad (3)$$

where ω is the natural frequency of oscillation of that vibrating body, which means, this is the frequency of oscillation of the freely oscillating body, and *G* is the damping, which generally depends on the viscosity (*b*) of the medium in which body oscillates.

After inserting in the differential equation it becomes

$$\frac{d^2x}{dt^2} + 2G\frac{dx}{dt} + \omega_o^2 x = 0 \qquad (4)$$

In the underdamped case: $\omega_o > G$

Since a damped oscillation tends to vanish along time, it can be assumed that the solution of equation (4) has the general form,

$$x(t) = A\,e^{-Gt}\,Sin(\,\omega t + \,p)$$

II. The Logarithmic Decrement (λ)

The (λ) Logarithmic Decrement is based on the assumption that the reduction of the amplitude in a damped oscillation is given by

$$\frac{x_{n+1}}{x_n} = \;e^{-\lambda t}$$

Then after applying logarithms to both sides:

$$\lambda = \;-\frac{1}{T}\ln\left[\frac{x_{n+1}}{x_n}\right]$$

The period of oscillation (*T*) in above equation is the time period of the damped oscillations, $T=2\pi/\omega$. For the sake of accuracy, the value of λ in equation above is obtained experimentally by averaging many instances of this equation.

III. Some practical applications of the Logarithmic Decrement

The Logarithmic Decrement has been applied in civil engineering and in mechanical spectroscopy of solids:

Magalas [4,5] reports the creation of a new technology to find the (λ)with high precision. This researcher introduces the Optimization in Multiple Intervals algorithm (OMI), which is approved for computations when the damping is rather high.

Butterworth et all [6] present an application of the logarithmic decrement in civil engineering, specifically to assess the dynamic response of structures (buildings) to vibrations due to earthquakes.

Majewski and Magalas [7] make use of the λ in the field of internal friction and mechanical spectroscopy of solids. In their paper these authors evaluate the application of the Hilbert transform to detect the envelope of damped oscillations with the aim on computing the Logarithmic Decrement.

IV. Comparative Study and Analysis

A comparison of readings of following three research papers was carried out and analyzed, hence results were plotted:

[1]. EduVirtualLab Implementing the Logarithmic Decrement to Identify the Parameters of an Oscillation :

This paper reports the development of a university-level EduVirtualLab (educational virtual lab) that in order to identify the control parameters (viscosity, initial phase, maximum amplitude) of an oscillation, applies the technique of Logarithmic Decrement, LD. Once the user of the virtual lab supplies the input data of the oscillation, the module displays the corresponding curve of displacement vs time, x(t), on computer screen. With the aim on obtaining reference theoretical results, the module automatically detects the extreme displacements (peaks and valleys) of the x(t) curve and it applies the LD algorithm to these data to calculate the damping of the system and, from this the control parameters of the oscillation are identified. Next the module allows the user to click with the mouse on the peaks (or valleys) of x(t) and, once the user has clicked 10 of these, the module computes the LD and from this, the experimental control parameters of the oscillation are obtained. The user may repeat this stage by clicking 10 valleys (or peaks) of x(t).

[2]. Use of the Logarithmic Decrement to Assess the Damping in Oscillations :

A Virtual Lab to experimentally detect the frequency of the oscillations of a damped oscillator has been created. At the end of the simulation, the virtual lab displays both, theoretical as well as experimental results, so that the user can compare them. In this way, if the user has correctly clicked the extremes (peaks and/or valleys) of x(t), his experimental results –as it is expected- are verified as being very close to the theoretical results calculated by the virtual lab.

[3]. Virtual Lab to Run Logarithmic Damping Decrement Experiments :

A university-level educational Virtual Lab that in order to detect the damping an oscillator is experimenting applies the technique of Logarithmic Damping Decrement has been created. After input parameters (mass, elastic constant and viscosity of the medium) are entered by the user of the simulation module, this displays the corresponding curve of displacement vs time, x(t), on computer screen. Next the module allows the user to manually click with the mouse on the peaks (or valleys) of the displayed curve and, once the user has clicked 10 of these, the module computes the logarithmic decrement and from this, the experimental damping of the oscillator. The user may repeat this stage by clicking 10 valleys (or peaks) of the x(t) curve. With the aim on obtaining reference theoretical results, right after input data has been entered the module automatically detects the extreme displacements of the x(t) curve and it applies the logarithmic decrement algorithm to these data and from this the damping of the system is calculated. At the end of the simulation the virtual lab shows the theoretical as well as the experimental results so that the user can compare them. In this way, if the user has correctly clicked the extremes (peaks and/or valleys) of x(t), his experimental results –as it is expected- are verified as being very close to the expected result calculated by the virtual lab.

As per comparison, following table was generated:

Table 1: *Comparison of data and results*

Criteria	***Paper 1***	***Paper 2***	***Paper 3***
mass	2 kg	4 kg	7 kg
elastic constant	83 N/m	70 N/m	34 N/m
Viscosity	0.35	0.84	0.55
Amplitude	84 deg	68 deg	90 deg
Results			
Damping of oscillator	0.0875	0.105	0.0393
Natural frequency	6.4420 rad/s	4.1833 rad/s	2.2039 rad/s
Damped frequency	6.4415 rad/s	4.1820 rad/s	2.2035 rad/s
Damped oscillations period	0.9754 sec	1.5024 sec	2.8514 sec
Mean λ	0.0863	0.105	0.0393

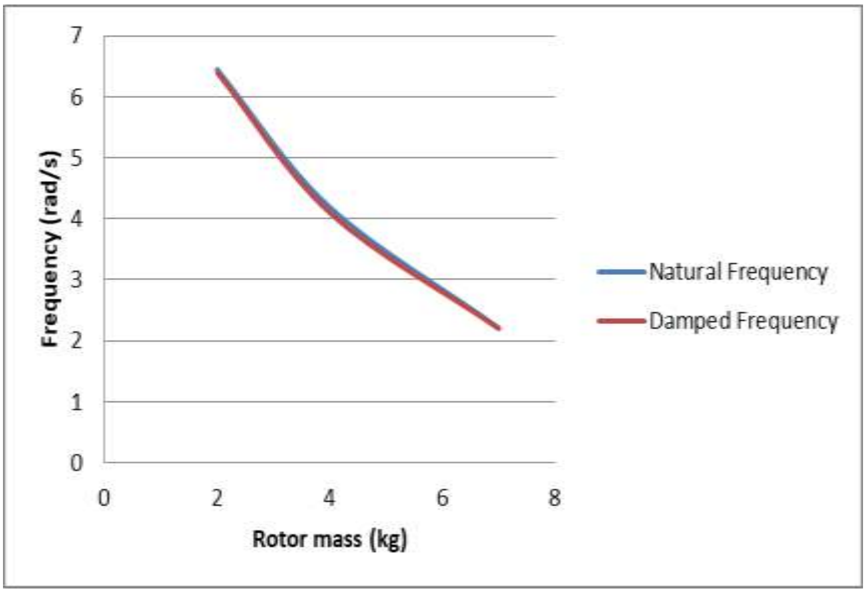

Fig.2: *Rotor Mass vs Frequency*

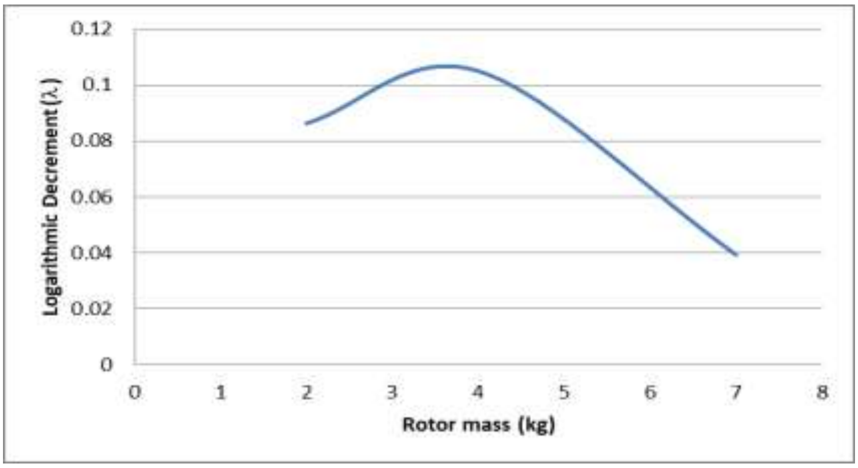

Fig.3: *Rotor Mass vs Logarithmic Decrement*

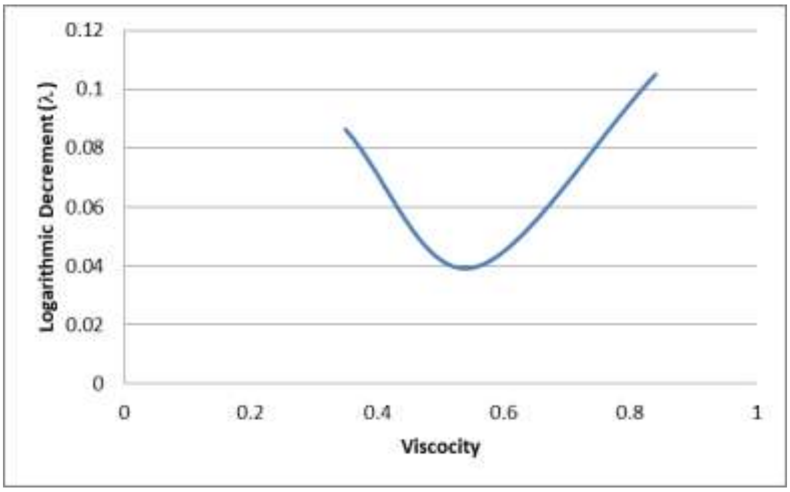

Fig.4: *Viscosity vs Logarithmic Decrement*

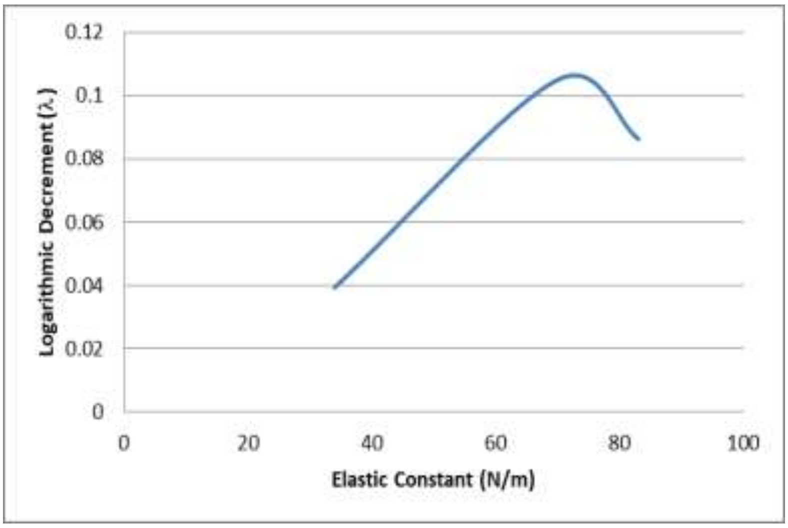

Fig.5: *Elastic Constant vs Logarithmic Decrtement*

After comparison of data, following results are evaluated:
(1). From Fig.2,3,4, and 5 we can state that Logarithmic decrement (λ) is dependent of mass of body, elastic constant of spring or shaft, viscosity of damping medium and is independent of initial Amplitude of vibration.
(2). From Fig.2 it is clear that Logarithmic decrement (λ) is directly proportional to mass of the body under oscillation upto certain limit and decreases gradually.
(3). Lambda (λ) is inversely proportional to elastic constant of spring or shaft due to which the body vibrates.
(4). As the viscosity increases, the Logarithmic Decrement of the system also increases.
(5). Natural Frequency of system is directly proportional to stiffness of the spring or shaft and inversely proportional to mass of the vibrating body.

V. Conclusion

In this paper, a review on computation of logarithmic decrement is presented. Using a virtual vibration lab software, systems were analyzed under particular damping condition. The process was followed by finding the behavior of the amplitude of the system due to damping. This behavior of reduction in amplitude called as Logarithmic decrement is calculated. After studying these virtual experiments, results were plotted and evaluated. Logarithmic decrement (λ) is dependent of mass of body, elastic constant of spring or shaft, viscosity of damping medium and is independent of initial Amplitude of vibration.

References

[1]. EduVirtualLab Implementing the Logarithmic Decrement to Identify the Parameters of an Oscillation , Javier Montenegro Joo ,VirtualDynamics: Science & Engineering Virtual Labs.

[2]. Use of the Logarithmic Decrement to Assess the Damping in Oscillations , Javier Montenegro Joo, VirtualDynamics: Science & Engineering Virtual Labs

[3]. Virtual Lab to Run Logarithmic Damping Decrement Experiments Javier Montenegro Joo Virtual Dynamics: Science & Engineering Virtual Labs

[4]. Magalas L.B., (2006) Determination of the Logarithmic Decrement in Mechanical Spectroscopy, Solid State Phenomena Vol 115.

[5]. Magalas L.B., Majewski M., (2008) Recent Advances in Determination of the Logarithmic Decrement and the Resonant Frequency in Low-Freq. Mech. Spectroscopy, Solid State Phenomena Vol. 137.

[6]. Butterworth J., Jin Hee Lee, Davidson B., (2004) Experimental determination of modal damping from full scale testing, 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada August 1-6, 2004, Paper No. 310.

[7]. Majewski M., Magalas L.B., (2015), Critical assessment of the issues in the application of Hilbert transform to compute the logarithmic decrement, Archives of Metalurgy and Materials 60(2).